III Cantor's faults in his doctrine of the transfinite

April 2023 Existence of the transfinite is disproved by article II. Hypothetically presuming existence, Cantor's works is scrutinised. The antagonistic disparity of transfinite ordinal and cardinal numbers is rebutted, the continuum hypothesis is demonstrated to be wrong. The ostensible equality of sets of points of extremely different geometrical objects, such as a small segment and n-dimensional space, is falsified. The paradox of equivalent transfinite sets and subsets, seemingly suspending Euclid's 5. axiom, "the whole is greater than the part", is resolved. Euclid's axiom is safeguarded.

1. Introduction

The transfinite ordinalnumber ω of the set ℕ of natural number as required by the axiomatic system ZFC and by Cantor's doctrine are rejected by article II [1]. The discussion of propositions based on these terms is actually superfluous. In the sense of an overview of Cantor's works the most important statements are nevertheless considered.
The discussion requires the distinction between **cardinal and ordinal numbers.** The ordinal number defines the ranking of the elements of an ordered set, e.g. the set { 1, 2, 3, …. n } with the largest element n owns the ordinal number n. The cardinal number describes the number of elements also in unordered sets. In the finite, cardinal and ordinal numbers always coincide. This also applies to the smallest transfinite numbers, the ordinal number ω of the set ℕ of natural numbers, and their cardinal number ℵo (Aleph 0). The equality no longer holds for larger transfinite numbers, they **show an antagonistic difference**. In the case of a single further transfinite element, the following applies:
(1) ℵo = ℵo + 1 but ω ≠ ω+1
These numbers are followed by other equal cardinal but unequal ordinal numbers. One must make this disparity clear to oneself in all its consequences. A further element requires a larger ordinal number, but the cardinal number is demanded to remain the same. **Cantor verifies equal cardinal numbers of two sets by a bijection**, the mutually unique relationship of the elements, that is meant to justify ℵo = ℵo + 1.

2. The bijection, meant to justify ℵo = ℵo + 1, is rebutted

Cantor justifies (1) for the set of natural numbers, { ν }, by "that ….. if a new element eo is added to the set { ν }, the set union { { ν }, eo } is equivalent to the original one{ ν }. Because the mutually unique relationship between the two can be imagined, according to which element eo of the first corresponds to element 1 of the second, element ν of the first corresponds to element ν + 1 of the other. ..... [therefore] we have ℵo = ℵo + 1."
The graphical representation allows the inspection, at first restricted to finite sets:

(first set) { { 1 2 3 4 ….................................. ν - 1 **ν** } eo }

(second set) { 1 2 3 4 5 .... ν -1 ν }

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The last number of the first set, ν, remains without assignment to an element of the second set. We can extend the two sequences of numbers and the mapping infinitely, as Cantor presumes, an unassigned element of the first set always remains . **There is never a bijection.**
A bijection to be excluded is finally demonstrated for potentially infinite elements.
The cardinal number does not depend on the kind of assignment of the elements. Below identical numbers are mapped to each other, also e_o of the first and 1 of the second set.

(first set) { { 1 2 3 4 5 .... ν-1 ν .... } e_o }
| | | | | | | | | | | | | |
(second set) { 1 2 3 4 5 .... ν-1 ν .... }

The first umber of the first set, 1, remains without mapping. Equivalence of the two sets is rebutted.
(2) $\aleph o = \aleph o + 1$ cannot be justified, $\aleph o \neq \aleph o + 1$ as well as $\omega \neq \omega+1$ applies.

3. The continuum hypothesis is wrong

We continue to follow Cantor's reasoning. Accordingly, not only $\aleph o = \aleph o + 1$ applies, but the equality of many other cardinal numbers is derived. Adding 1 to both sides results in (3):
(3) $\aleph o + 1 = \aleph o + 2$ and thus $\aleph o = \aleph o + 1 = \aleph o + 2$
The sequence of equal cardinal numbers continues until $\aleph o^n$.
(4) $\aleph o = \aleph o + 1 = \aleph o + \aleph o = n\ \aleph o = \aleph o \cdot \aleph o = \aleph o^n$
Even the rational numbers, which in addition to many fractions of whole numbers contain comparatively few natural numbers, can nevertheless be counted by natural numbers, as Cantor demonstrates with his first diagonal argument. **The rational numbers also own the cardinal number $\aleph o$. But for the cardinal number of the set of real numbers[2], according to Cantor $2^{\aleph 0}$, he shows** with his 2nd diagonal argument:
(5) $2^{\aleph 0} > \aleph o$
The assumption that there are no other cardinal numbers between $\aleph o$ and $2^{\aleph 0}$ is called 'Cantor's continuum hypothesis'. It was established in 1878. David Hilbert declared it the most important unsolved problem in mathematics at the International Congress of Mathematicians in Paris in 1900. It occupied scientists for many decades. Kurt Gödel demonstrated in 1938 that it cannot be refuted from the axiomatic system ZFC, Paul Cohen showed that it cannot be proved from it and was awarded the Fields Medal for this in 1966. The continuum hypothesis is undecidable from the axioms. Subsequently will be shown that Cantor's justification of the hypothesis is wrong. **Cantor uses formula (6) to calculate the real numbers x.**
(6) $x = f(\nu) / 2 + f(\nu) / 2^2 + \dots f(\nu) / 2^n + \dots$ ($f(\nu) = 0$ or 1)
The set of numbers x can be explained by continued halving of the interval [0, 1] and of the resulting halves **The cardinal number of { x } therefore is $2^{\aleph 0}$.**[3] But numbers x, generated by (6), contain only natural numbers in the numerator and only powers of 2 in the denominator, i.e. always form integer fractions. So **formula (6) for calculating all real numbers generates only rational numbers.** According to Cantor's 1st diagonal argument, their cardinal number is equal to $\aleph o$.
(7) $2^{\aleph 0} = \aleph o$
There is a contradiction to (5). In the book "Nothing" it is shown that the 2nd diagonal argument is also only based on rational numbers, i.e. (5) cannot be justified by it.
The basis (5) of the continuum hypothesis is wrong, (7) is valid.

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2 The real numbers consist of the rational and the irrational numbers (limit values of infinite converging sequences of rational numbers).

3 Inserting 1 in the first term and 0 in all other terms results in number 1/2. 1 in the second term and 0 in all others yields 1/4, numbers 1/8 can be calculated accordingly. The terms are completed by additions according to (6).

4. Equality of sets of points of different geometric objects is falsified

In addition to (4), **Cantor uses the power rules to calculate other identical cardinal numbers,** where $2^{\aleph 0} \equiv \aleph_1$ is set:

(8) $\aleph_1 = \aleph 1^2 = \aleph_1^3 = \ldots\ldots = \aleph_1^n = \aleph_1^{\aleph 0}$ [4]

From the equality of these numbers, Cantor drew a conclusion that he himself characterized as 'incredible'. "Je le vois, mais je ne le croi pas", "I see it but I can't believe it".

He assumes that the cardinal numbers can be mapped to points. $\aleph_1$ is then the point set on the segment [0, 1], $\aleph_1^2$ defines the point set of a square, $\aleph_1^n$ the set in an n-dimensional and $\aleph_1^{\aleph 0}$ in an ℵo-dimensional structure. **According to (8), all these sets of points are equal.**

But Cantor contradicts himself with this. In his justification of ℵo = ℵo + 1 he demands that, despite the same cardinal number, there is another element or point. Additional points thus leave the cardinal number the same, but larger ordinal numbers are formed. This is shown by Cantor's sequence of ordinal numbers, which keeps getting bigger despite of the same cardinal numbers:

(10) 1, 2, 3, ...ν ... ω, ω+1, ω+2, ... 2ω, 2ω+1, ... 3ω,.... .ω2,.... ω3, ..., ω^{ω} , …............

The ordinal numbers determine the sets of points, not the cardinal numbers, as Cantor assumes. **The alleged equality of point sets of different geometric objects does not exist.**

5. Cantor's misinterpretation of the bijection of transfinite sets and subsets

The equivalence of sets and subsets is the most characteristic property of the transfinite for Cantor, in contradiction to Euclid's 5th axiom, "the whole is greater than the part".

The cardinal number of equivalent sets and subsets is the same. The natural numbers, square numbers and even numbers are given as an example:

1	2	3	4	5	6 …...........................	n	…...............................
1^2	2^2	3^2	4^2	5^2	6^2 …....................	n^2	…...............................
2	4	6	8	10	12….........................	2n	…................................

The equivalence of natural numbers and square numbers in infinity had already worried Galileo and went down in the history of mathematics as "Galileo's paradox".

In fact, the equivalence already exists in the finite. Different densities of the elements of various sequences are responsible for the apparent paradox, as shown for finite sequences of natural and even numbers.

(a)	1	2	3	4	5	6	n
(b)		2		4		6 …...........................	n
(c)	2	4	6	8	10	12 …........................	2n

The seemingly paradox equivalence of (a) and (c) results from different densities of the even numbers. These of sequences (a) and (b) however own the same density. The cardinal number of the even numbers (b) is only half as large as that of the natural numbers (a), the cardinal numbers of (a) and (c) yet are the same without insinuating paradoxical properties. . All other equivalences are explained analogously.

Euclid's "The whole is greater than the part", remains valid.

4 The next larger cardinal number is given by $\aleph_2 = 2^{\aleph 1}$, another one by $\aleph_3 = 2^{\aleph 2}$ etc.
(9) 1, 2, 3, 4, 5, ...n, ... ℵo, $2^{\aleph 0}$, $2^{\aleph 1}$, $2^{\aleph 2}$, $2^{\aleph 3}$…...................…
Cantor demands continuous levels of infinity for the cardinal numbers.